Listed Volatility and Variance Derivatives
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Contents

Preface xi

PART ONE Introduction to Volatility and Variance

CHAPTER 1 Derivatives, Volatility and Variance 3
1.1 Option Pricing and Hedging 3
1.2 Notions of Volatility and Variance 6
1.3 Listed Volatility and Variance Derivatives 7
   1.3.1 The US History 7
   1.3.2 The European History 8
   1.3.3 Volatility of Volatility Indexes 9
   1.3.4 Products Covered in this Book 10
1.4 Volatility and Variance Trading 11
   1.4.1 Volatility Trading 11
   1.4.2 Variance Trading 13
1.5 Python as Our Tool of Choice 14
1.6 Quick Guide Through the Rest of the Book 14

CHAPTER 2 Introduction to Python 17
2.1 Python Basics 17
   2.1.1 Data Types 17
   2.1.2 Data Structures 20
   2.1.3 Control Structures 22
   2.1.4 Special Python Idioms 23
2.2 NumPy 28
2.3 matplotlib 34
2.4 pandas 38
   2.4.1 pandas DataFrame class 39
   2.4.2 Input-Output Operations 45
   2.4.3 Financial Analytics Examples 47
2.5 Conclusions 53
## CONTENTS

### CHAPTER 3

**Model-Free Replication of Variance**

- **3.1 Introduction**
- **3.2 Spanning with Options**
- **3.3 Log Contracts**
- **3.4 Static Replication of Realized Variance and Variance Swaps**
- **3.5 Constant Dollar Gamma Derivatives and Portfolios**
- **3.6 Practical Replication of Realized Variance**
- **3.7 VSTOXX as Volatility Index**
- **3.8 Conclusions**

### PART TWO

**Listed Volatility Derivatives**

### CHAPTER 4

**Data Analysis and Strategies**

- **4.1 Introduction**
- **4.2 Retrieving Base Data**
  - 4.2.1 EURO STOXX 50 Data
  - 4.2.2 VSTOXX Data
  - 4.2.3 Combining the Data Sets
  - 4.2.4 Saving the Data
- **4.3 Basic Data Analysis**
- **4.4 Correlation Analysis**
- **4.5 Constant Proportion Investment Strategies**
- **4.6 Conclusions**

### CHAPTER 5

**VSTOXX Index**

- **5.1 Introduction**
- **5.2 Collecting Option Data**
- **5.3 Calculating the Sub-Indexes**
  - 5.3.1 The Algorithm
- **5.4 Calculating the VSTOXX Index**
- **5.5 Conclusions**
- **5.6 Python Scripts**
  - 5.6.1 index_collect_option_data.py
  - 5.6.2 index_subindex_calculation.py
  - 5.6.3 index_vstoxx_calculation.py

### CHAPTER 6

**Valuing Volatility Derivatives**

- **6.1 Introduction**
- **6.2 The Valuation Framework**
- **6.3 The Futures Pricing Formula**
PART THREE

Listed Variance Derivatives

CHAPTER 9
Realized Variance and Variance Swaps

9.1 Introduction

9.2 Realized Variance

9.3 Variance Swaps
  9.3.1 Definition of a Variance Swap
  9.3.2 Numerical Example
  9.3.3 Mark-to-Market
  9.3.4 Vega Sensitivity
  9.3.5 Variance Swap on the EURO STOXX 50

9.4 Variance vs. Volatility
  9.4.1 Squared Variations
  9.4.2 Additivity in Time
  9.4.3 Static Hedges
  9.4.4 Broad Measure of Risk

9.5 Conclusions

CHAPTER 10
Variance Futures at Eurex

10.1 Introduction

10.2 Variance Futures Concepts
  10.2.1 Realized Variance
  10.2.2 Net Present Value Concepts
  10.2.3 Traded Variance Strike
  10.2.4 Traded Futures Price
  10.2.5 Number of Futures
  10.2.6 Par Variance Strike
  10.2.7 Futures Settlement Price

10.3 Example Calculation for a Variance Future

10.4 Comparison of Variance Swap and Future

10.5 Conclusions

CHAPTER 11
Trading and Settlement

11.1 Introduction

11.2 Overview of Variance Futures Terms

11.3 Intraday Trading

11.4 Trade Matching

11.5 Different Traded Volatilities

11.6 After the Trade Matching

11.7 Further Details
  11.7.1 Interest Rate Calculation
  11.7.2 Market Disruption Events

11.8 Conclusions
## PART FOUR

**DX Analytics**

### CHAPTER 12

**DX Analytics – An Overview**

12.1 Introduction 283
12.2 Modeling Risk Factors 284
12.3 Modeling Derivatives 287
12.4 Derivatives Portfolios 290
   12.4.1 Modeling Portfolios 292
   12.4.2 Simulation and Valuation 293
   12.4.3 Risk Reports 294
12.5 Conclusions 296

### CHAPTER 13

**DX Analytics – Square-Root Diffusion**

13.1 Introduction 297
13.2 Data Import and Selection 297
13.3 Modeling the VSTOXX Options 301
13.4 Calibration of the VSTOXX Model 303
13.5 Conclusions 308
13.6 Python Scripts 308
   13.6.1 dx_srd_calibration.py 308

### CHAPTER 14

**DX Analytics – Square-Root Jump Diffusion**

14.1 Introduction 315
14.2 Modeling the VSTOXX Options 315
14.3 Calibration of the VSTOXX Model 320
14.4 Calibration Results 325
   14.4.1 Calibration to One Maturity 325
   14.4.2 Calibration to Two Maturities 325
   14.4.3 Calibration to Five Maturities 325
   14.4.4 Calibration without Penalties 331
14.5 Conclusions 332
14.6 Python Scripts 334
   14.6.1 dx_srjd_calibration.py 334

**Bibliography**

**Index**
Volatility and variance trading has evolved from something opaque to a standard tool in today’s financial markets. The motives for trading volatility and variance as an asset class of its own are numerous. Among others, it allows for effective option and equity portfolio hedging and risk management as well as straight out speculation on future volatility (index) movements. The potential benefits of volatility- and variance-based strategies are widely accepted by researchers and practitioners alike.

With regard to products it mainly started out around 1993 with over-the-counter (OTC) variance swaps. At about the same time, the Chicago Board Options Exchange introduced the VIX volatility index. This index still serves today – after a significant change in its methodology – as the underlying risk factor for some of the most liquidly traded listed derivatives in this area. The listing of such derivatives allows for a more standardized, cost efficient and transparent approach to volatility and variance trading.

This book covers some of the most important listed volatility and variance derivatives with a focus on products provided by Eurex. Larger parts of the content are based on the Eurex Advanced Services tutorial series which use Python to illustrate the main concepts of volatility and variance products. I am grateful that Eurex allowed me to use the contents of the tutorial series freely for this book.

Python has become not only one of the most widely used programming languages but also one of the major technology platforms in the financial industry. It is more like a platform since the Python ecosystem provides a wealth of powerful libraries and packages useful for financial analytics and application building. It also integrates well with many other technologies, like the statistical programming language R, used in the financial industry. You can find links to all Python resources under http://lvvd.tpq.io.

I thank Michael Schwed for providing parts of the Python code. I also thank my family for all their love and support over the years, especially my wife Sandra and our children Lilli and Henry. I dedicate this book to my beloved dog Jil. I miss you.

YVES
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